Exact solution to the conjugate problem of nonuniform cooling of fuel rods

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In this article, the analytical solution to a conjugate heat transfer problem is presented. The temperature distribution of the cladding of a fuel rod is determined, assuming that the internal heat generation rate is constant, while the local heat transfer coefficient is variable along the cladding perimeter, because of contact between adjacent rods. The contact occurs in one point (four-cusp channel) or along a line of the wetted perimeter.

Due to asymmetric geometry, the heat transfer coefficient depends on the blocking percentage of the channel and vanishes at the points of contact between adjacent rods. The energy balance equation is solved in two regions ($h=0$ in the former, and h given by a quadratic form in the latter) of the rod perimeter. This quadratic form was deduced by Turner et al. in 1982, solving numerically the fluid-flow problem. The solution of the thermal problem is obtained resorting to the use of Green's function; the results are given in terms of parabolic-cylinder functions.

Some graphs are obtained and discussed; the results show satisfactory agreement with other data available in the literature. Numerical work was performed by personal computer.

Keywords: nonuniform cooling; fuel rods

Introduction

The cooling of tube or rod bundles occurs in numerous engineering applications and represents an involved application of thermal science.

Manufacturing tolerances, blockages, deformations, and asymmetrical bundle geometries have a strong influence on the flow and temperature distribution, and give rise to significant differences in the wall temperature on the circumference of tubes or rods, which determines additional bowing, swelling, and stresses.

The determination of the wall temperature under these anomalous conditions is important mainly in nuclear engineering. In fact, in the event of a loss-of-coolant accident (LOCA) in a pressurized water reactor (PWR), the lack of cooling leads to increased temperature levels in the fuel rod bundle, and the pressure of the gas in the gap (between fuel and cladding) will greatly exceed the coolant pressure.

These combined thermal and mechanical effects cause swelling of the cladding (ballooning) to such an extent that adjacent rods can make contact, reducing the coolant flow area and worsening the cooling of the local cladding.[1–4]

The determination of the wall temperature under these ballooned conditions is very important in predicting circumferential gradients and hot spots near lines of contact for the bundle. The solution to such a problem can be of great concern in the context of safety analysis and licensing procedures, in order to provide evidence that all the consequences of an accident can be controlled.

At the present time, the computer codes used for the safety analysis of nuclear power plants (TRAC by Los Alamos National Laboratory, CHATARE by CEN Grenoble, RELAP by EG&G Idaho National Laboratory) do not take into account the consequences of ballooning, considering only symmetrical geometries for the rod bundle. Therefore, it is appropriate to propose analytical or numerical methods to calculate the azimuthal temperature variations on the cladding surface. As a contribution to the solution of this problem, various researchers[5–7] have made a flow and heat transfer simulation in a four-cusp channel, in order to predict large azimuthal variations of the local heat transfer coefficient.

The cladding temperature distribution is generally determined by numerical techniques. Only recently have analytical works been carried out[8–9] resorting to a conjugate model, with the heat transfer coefficient varying according to linear or quadratic expressions with respect to the azimuthal coordinate in a deformed channel geometry.

The aim of this article is to contribute to the investigation of the heat transfer problem; in the cladding of deformed channels, the energy balance equation is solved analytically, considering a heat transfer coefficient distribution similar to the one proposed in Figure 6 of ref. 6, which was deduced by solving numerically the fluid-flow problem.

In this connection, it is useful to introduce the blocking percentage P, defined as the ratio between the actual coolant cross section and the unblocked flow area. Referring to a typical PWR fuel rod bundle with a pitch and pellet diameter equal to 12.6 and 9.5 mm, respectively, the percentage blocking is equal to 61.23% if contact between adjacent rods occurs at a single point (four-cusp channel). If $P<61.23\%$, there is no contact between adjacent rods, while if $P>61.23\%$ the contact occurs on a line, generating an adiabatic region on the cladding surface. In this work a percentage blocking equal to or greater than 61.23% is considered.

This problem, typical of a pressurized water reactor, is present in other engineering applications, for example, in heat exchangers and fusion reactors, where the surface heat flux on the first wall and on the limiter/divertor plate varies circumferentially.[10]

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Received 20 November 1989; accepted 19 November 1990

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Mathematical model

Considering a control volume in the cladding, the general energy balance equation can be written as

$$ks\frac{d^2t}{dy^2}-h(y)(t-t_b)+q=0 \qquad (1)$$

Equation 1 represents a conjugate model based on the following hypotheses:

(1) the eccentricity of the fuel pellets is neglected;
(2) the radial and axial gradients of cladding temperature are negligible;
(3) the effect of curvature on the temperature distribution is negligible;
(4) the heat flux at the fuel–cladding interface is constant with the azimuthal coordinate; this hypothesis agrees with the results of Sdouz and Dagbjartsson;[4]
(5) the physical properties of materials are constant;
(6) the diameter of the swollen rods coincides with the pitch of the rods;
(7) steady-state conditions.

The coolant bulk temperature is constant and the heat transfer coefficient depends on the azimuthal coordinate.

Haque et al.[6] have numerically determined the local variation of the heat transfer coefficient with the azimuthal coordinate, in laminar and turbulent flow. Referring to a unidirectional model, the mathematical form chosen for the heat transfer coefficient is

$$h(y)=\begin{cases} 0 & \text{if } 0\le y/L\le f \\ -A+B(y/L)-C(y/L)^2 & \text{if } f\le y/L\le 1 \end{cases} \qquad (2)$$

Expressions for the constants A, B, C are

$$A=\frac{f(2-f)}{(1-f)^2}h_0, \qquad B=\frac{2}{(1-f)^2}h_0, \qquad C=\frac{2}{(1-f)^2}h_0$$

According to ref. 5, in the four-cusp channel: $f\simeq 0.25$, $h_0\simeq 2\langle h\rangle$. In Figure 1, the heat transfer coefficient, given by Equation 2, is plotted versus the azimuthal coordinate.

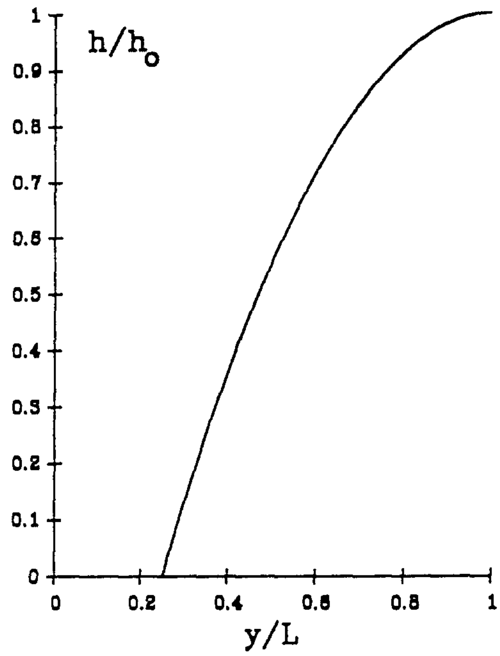

Figure 1 Local heat transfer coefficient versus the azimuthal coordinate on the cladding surface

Solution to the heat balance equation

Substituting Equation 2 into Equation 1, two different equations are obtained for the regions $0\le y/L/f$ and $f\le y/L\le 1$.

It is useful to introduce the following quantities:

$$\text{Bi}=\frac{h_0 s}{k}, \qquad \mu=\frac{s}{L}, \qquad M=\frac{qL}{(4Cks)^{1/2}},$$

$$T=\frac{1}{M}(t-t_b), \qquad E=\left[\frac{4CL^2}{ks}\right]^{1/4}=[4\text{Bi}/\mu^2]^{1/4}[1-f]^{-1/2}, \qquad (3)$$

$$a=\left[\frac{(1-f)E}{2}\right]^2, \qquad x=E(1-y/L)$$

Notation

a	Dimensionless parameter $[(1-f)E]^2/4$
Bi	Biot number at the point B (see Figure 2)
c	Cladding specific heat (J/kg K)
E	Dimensionless parameter
f	Unwetted fraction of the AB perimeter (see Figure 2)
$G(\cdot)$	Green's function
$h(\cdot)$	Coolant-cladding local heat transfer coefficient (W/m^2 K)
h_0	Maximum coolant-cladding heat transfer coefficient (W/m^2 K)
$\langle h\rangle$	Average coolant-cladding heat transfer coefficient (W/m^2 K)
k	Cladding thermal conductivity (W/m K)
L	Length of the AB perimeter (m) (see Figure 2)
M	Dimensional parameter (K)
P	Percentage blocking (%)
p	Auxiliary dimensionless coordinate
q	Heat flux at the fuel–cladding interface (W/m^2)
s	Cladding thickness (m)
$t(\cdot)$	Cladding temperature (K)
t_b	Coolant bulk temperature (K)
$T(\cdot)$	Dimensionless cladding temperature
$W_j(\cdot)$	Parabolic-cylinder functions ($j=1,2$)
x	Dimensionless curvilinear coordinate
y	Curvilinear coordinate (m)
Greek symbols	
$\delta(\cdot)$	Dirac delta function
μ	Thickness-to-length ratio (s/L)
ρ	Cladding density (kg/m^3)
τ	Time (s)
ω	Dimensionless parameter $(1-f)E$
Subscripts	
I, II	First and second domain on the rod perimeter, respectively

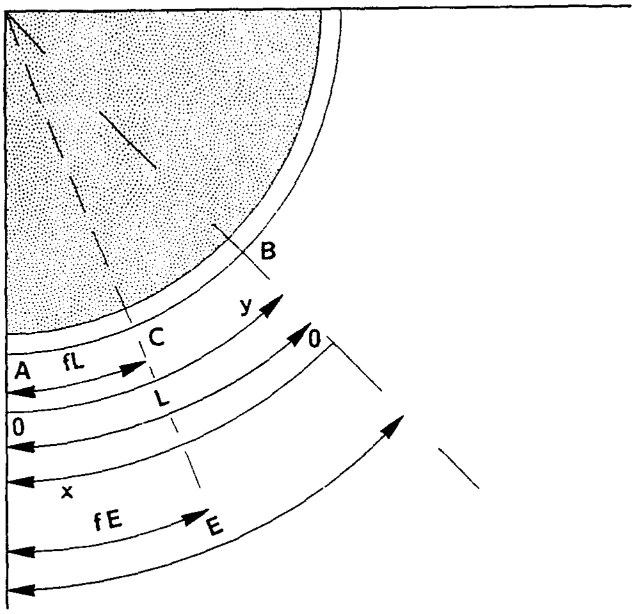

Figure 2 View of the cross section of the fuel rod, indicating the *x, y* coordinates, the points A, B, C, and the length of the AC and AB segments

where Bi represents the usual Biot number at point B, and μ is the dimensionless thickness of the cladding. In Figure 2, the geometrical connection between x and y is clearly shown. The dimensionless conjugate heat balance equation then becomes

$$\frac{d^2T}{dx^2}+\left(\frac{x^2}{4}-a\right)T+1=0 \qquad 0\le x\le(1-f)E \tag{4}$$

$$\frac{d^2T}{dx^2}+1=0 \qquad (1-f)E\le x\le E \tag{5}$$

for $f\le y/L\le 1$ and $0\le y/L\le f$, respectively.

Referring to Figure 2, the boundary conditions that the temperature distribution must satisfy are

at point A:

$$\frac{dT}{dx}=0 \qquad \text{because of symmetry;} \tag{6a}$$

at point C:

$$\frac{dT}{dx}=fE \qquad \text{energy balance;} \tag{6b}$$

$$T_{II}=T_I[x=(1-f)E] \qquad \text{continuity of temperature;} \tag{6c}$$

at point B:

$$\frac{dT}{dx}=0 \qquad \text{because of symmetry.} \tag{6d}$$

(a) $0\le x\le(1-f)E$

In this domain, the heat balance and the two boundary conditions read as

$$\frac{d^2T}{dx^2}+\left(\frac{x^2}{4}-a\right)T+1=0$$
$$\frac{dT}{dx}=0 \qquad \text{for } x=0 \tag{7}$$
$$\frac{dT}{dx}=fE \qquad \text{for } x=(1-f)E$$

Introducing Green's function $G(x, p)$, the dimensionless temperature can be expressed (see Appendix) as

$$T(x)=-\left\{\int_0^{(1-f)E} G(x,p)\,dp+fEG(x,p=(1-f)E)\right\} \tag{8}$$

where $p\in[0,(1-f)E]$ is an auxiliary dimensionless independent variable.

Green's function must satisfy the following conditions:

$$\frac{\partial^2G}{\partial x^2}+\left(\frac{x^2}{4}-a\right)G(x,p)=\delta(x-p) \tag{9a}$$

$$\frac{\partial G}{\partial x}=0 \qquad \text{for } x=0 \text{ and } x=(1-f)E \tag{9b}$$

where $\delta(x-p)$ is the Dirac delta function. Considering Equation 9a for $x\in[0,p[$ and $x\in]p,(1-f)E]$, we have $\delta(x-p)=0$. In this way we obtain a parabolic-cylinder equation equal in both domains. If $W_1(x)$ and $W_2(x)$ are two independent solutions that can be expressed by[11]

$$W_1(x)=1+a\frac{x^2}{2!}+\left(a^2-\frac{1}{2}\right)\frac{x^4}{4!}+\left(a^3-\frac{7}{2}a\right)\frac{x^6}{6!}+\cdots \tag{10a}$$

$$W_2(x)=x+a\frac{x^3}{3!}+\left(a^2-\frac{3}{2}\right)\frac{x^5}{5!}+\left(a^3-\frac{13}{2}a\right)\frac{x^7}{7!}+\cdots \tag{10b}$$

$G(x, p)$ can be expressed as

$$G(x,p)=\begin{cases}A_1(p)W_1(x)+A_2(p)W_2(x) & 0\le x\le p\\ A_3(p)W_1(x)+A_4(p)W_2(x) & p\le x\le(1-f)E\end{cases} \tag{11}$$

Hence the solution $T(x)$, Equation 8, is completely determined in the first domain $0\le x\le(1-f)E$.

(b) $(1-f)E\le x\le E$

The conjugate equation is now

$$\frac{d^2T}{dx^2}+1=0 \tag{12}$$

associated with the boundary conditions 6b and 6c. The solution to such a problem is

$$T_{II}(x)=-\frac{x^2}{2}+Ex-\frac{1}{2}(1-f^2)E^2+T_I(x) \tag{13}$$

We observe that

$$\frac{dT_{II}}{dx}=fE \qquad \text{at } x=(1-f)E \tag{14}$$

Mathematically, this condition ensures continuity of the first derivative of $T(x)$ at point C; from the physical point of view, it also satisfies continuity of the thermal flux at the interface between the two domains. The whole mathematical procedure is described in more detail in the Appendix.

Results and conclusions

This section presents the results concerning the deformed fuel rod of a PWR, following a LOCA. Typical values of the physical and geometrical parameters are[5,8]

● average heat transfer coefficient (W/m^2 K)	300.00
● zircaloy thermal conductivity (W/m K)	19.00
● heat flux in the inner cladding (kW/m^2)	38.00
● pitch of the fuel rod (mm)	12.60
● cladding thickness (mm)	0.57
● diameter of the fuel rod (mm)	9.50

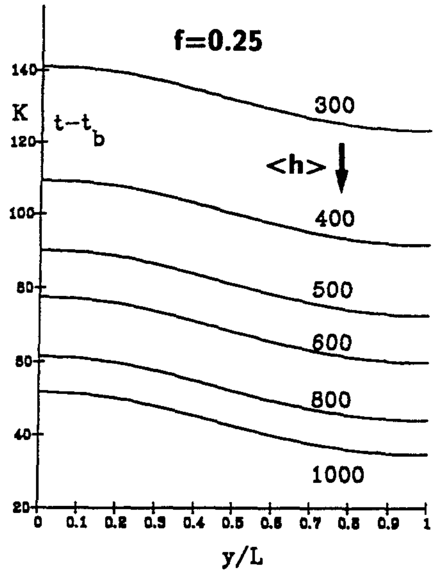

Figure 3 Wall temperature distribution versus the azimuthal coordinate, for various values of the average heat transfer coefficient and $f=0.25$

The temperature difference $t-t_b$ is analytically determined; as stated in Equations 3, it is directly proportional to the heat flux in the inner cladding and to the length of the perimeter considered, and inversely proportional to the square root of the cladding thermal conductivity and thickness. It is emphasized that the dimensionless temperature T depends only on two dimensionless parameters, f and E. The former is simply the unwetted fraction ($h=0$) of the cladding perimeter; the latter depends on the heat transfer conditions and the geometrical dimensions.

In Figure 3, the temperature difference distribution $t-t_b$ is plotted as a function of the dimensionless coordinate y/L, for different values of the average heat transfer coefficient, with $f=25\%$. The trend is very similar to that one found by Erbacher[1] for a different heat transfer coefficient distribution and for $f=0$, but for the same mean values. In particular, for $\langle h\rangle=300$ W/m^2 K, the graph shows $\Delta t_{max}=141.1$ K and $\Delta t_{min}=122.7$ K, where $\Delta t=t-t_b$. It is interesting to observe that for $\langle h\rangle$ ranging from 300 to 1000 W/m^2 K the difference $(\Delta t_{max}-\Delta t_{min})$ varies from 18.3 K to 17.4 K. This difference is slightly affected by the variations of the average heat transfer coefficient, while temperature is significantly affected. Moreover, by increasing $\langle h\rangle$, the temperature profiles tend to have an increasing slope.

Figure 4 shows the dimensionless temperature T as a function of the y/L coordinate, for f ranging from 0 to 25% and for $E=2$. It is evident that for an increase of f the temperature T rises; this is in agreement with physical perception. In particular, increasing f causes the temperature difference [T(0)−T(1)] to rise progressively; for example, for f equal to 15%, 20%, and 25% this difference is 0.627, 0.756, and 0.840, respectively. Decreasing f causes the minimum value of temperature to reduce more rapidly than the maximum value.

Finally, in Figure 5 the dimensionless temperature profile is plotted for different values of the E parameter, for $f=0$. The figure shows that by increasing E, the temperature T, calculated at point $y/L=0$, decreases more slowly than the temperature calculated at point $y/L=1$. Moreover, the difference [T(0)–T(1)] is an increasing function of E; for $E=1.75$ its value is 0.346, and for $E=2.5$ its value is 0.547.

Direct comparison with other published results is not possible, partly due to the different procedure for obtaining dimensionless temperature, but mainly due to the different trends of the local heat transfer coefficient. Elsewhere,[8,9] the problem is solved with other shapes of the function $h(x)$. However, comparison provides a reasonable indication of the cladding temperature; the more the local heat transfer coefficient increases, the lower are the temperatures. Figure 4 in ref. 5 shows the peripheral variation of temperature in the cladding surface for a four-cusp channel. In this case also, direct comparison with the results of this article is not possible, because the physical parameters are different; however, the qualitative trend is the same.

In conclusion, the presented analytical solution provides a fast and reliable tool to determine the temperature distribution in the cladding surface in deformed geometry with nonuniform cooling. It could be used also as a benchmark for more refined and onerous computer codes based on numerical solutions.

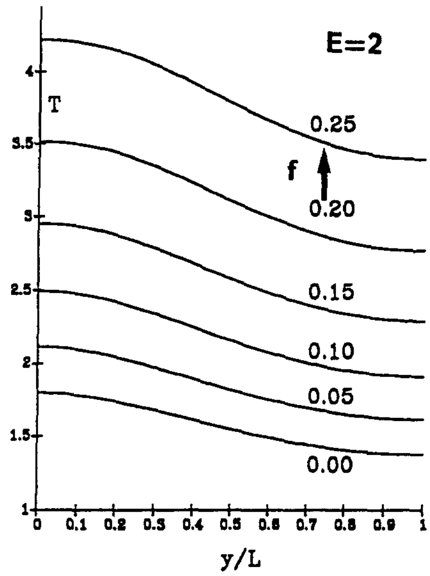

Figure 4 Dimensionless azimuthal temperature for various values of f, with $E=2$

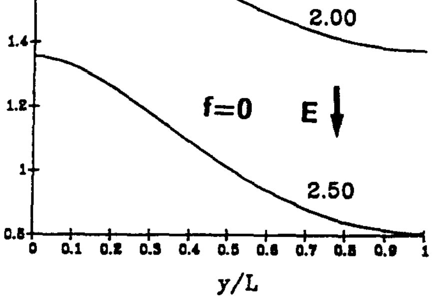

Figure 5 Dimensionless azimuthal temperature for various values of E, with $f=0$

Acknowledgments

This work was performed with financial support from the Italian MPI and CNR. Also the support of the National Group for Mathematical Physics (GNFM) is gratefully acknowledged.

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Appendix

If $G(x, p)$ solves the following boundary problem,

$$\frac{\partial^2 G}{\partial x^2}+\left(\frac{x^2}{4}-a\right)G(x,p)=\delta(x-p) \quad \text{(A.1)}$$

$$\frac{\partial G}{\partial x}=0 \qquad \text{for } x=0 \text{ and } x=(1-f)E \quad \text{(A.2)}$$

then the solution of the problem,

$$\frac{d^2 T}{dx^2}+\left(\frac{x^2}{4}-a\right)T+1=0 \quad \text{(A.3)}$$

$$\frac{dT}{dx}=0 \qquad \text{for } x=0 \quad \text{(A.4)}$$

$$\frac{dT}{dx}=fE \qquad \text{for } x=(1-f)E \quad \text{(A.5)}$$

can be obtained by means of Green's function $G(x, p)$. Multiplying Equation A.1 by the function $T(x)$ and Equation A.3 by $G(x, p)$, and subtracting the former from the latter, we obtain

$$\frac{\partial^2 G}{\partial x^2}T(x)-\frac{d^2 T}{dx^2}G(x,p)-G(x,p)=T(x)\,\delta(x-p) \quad \text{(A.6)}$$

This equation can be integrated with respect to x:

$$\int_0^\omega \frac{\partial^2 G}{\partial x^2}T(x)\,dx-\int_0^\omega \frac{d^2 T}{dx^2}G(x,p)\,dx-\int_0^\omega G(x,p)\,dx=T(p) \quad \text{(A.7)}$$

where $T(p)$ is the integral of $T(x)\,\delta(x-p)$ from 0 to ω, if $0<p<\omega$.[12] Integration by parts of the first two terms on the left-hand side gives

$$\frac{\partial G}{\partial x}T(x)\bigg|_0^\omega-\frac{dT}{dx}G(x,p)\bigg|_0^\omega-\int_0^\omega G(x,p)\,dx=T(p)$$

and hence,

$$\left.\frac{\partial G}{\partial x}\right|_\omega T(x)-\left.\frac{\partial G}{\partial x}\right|_0 T(0)-\left.\frac{dT}{dx}\right|_\omega G(x,p)+\left.\frac{dT}{dx}\right|_0 G(0,p) -\int_0^\omega G(x,p)\,dx=T(p) \quad \text{(A.8)}$$

The first and second terms are equal to zero because of Conditions A.2; the third and fourth are equal to $fEG(x=\omega, p)$ and 0, respectively, according to Conditions A.4 and A.5. Then, $T(p)$ is completely defined. Since p is a quantity that can vary from 0 to ω, and since $G(x, p)$ is symmetrical with respect to x and p, we can formally substitute x for p. Finally:

$$T(x)=-\left\{\int_0^{(1-f)E} G(x,p)\,dp+fEG(x,p=(1-f)E)\right\} \quad \text{(A.9)}$$